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Stretching the horizon of a higher dimensional small black hole

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Abstract: There is a general scaling argument that shows that the entropy of a small black hole, representing a half-BPS excitation of an elementary heterotic string in any dimension, agrees with the statistical entropy up to an overall numerical factor. We propose that for suitable choice of field variables the near horizon geometry of the black hole in D space-time dimensions takes the form of $AdS_2 \times S^{D-2}$ and demonstrate how this ansatz can be used to calculate the numerical factor in the expression for the black hole entropy if we know the higher derivative corrections to the action. We illustrate this by computing the entropy of these black holes in a theory where we modify the supergravity action by adding the Gauss-Bonnet term. The black hole entropy computed this way is finite and has the right dependence on the charges in accordance with the general scaling argument, but the overall numerical factor does not agree with that computed from the statistical entropy except for $D = 4$ and $D = 5$. This is not surprising in view of the fact that we do not use a fully supersymmetric action in our analysis; however this analysis demonstrates that higher derivative corrections are capable of stretching the horizon of a small black hole in arbitrary dimensions.

Keywords: Superstrings and Heterotic Strings, Black Holes in String Theory.

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Contents

1. Introduction and summary

For heterotic string compactification to space-time dimension $D \leq 9$ with at least $N = 2$ supersymmetry, we have a set of half-BPS states in the spectrum of elementary string states [1, 2]. The simplest examples of such states are elementary string states wound w times on a circle and carrying n units momentum along the same circle. For a fixed set of charge quantum numbers the degeneracy of these BPS states grow as $\exp(4\pi\sqrt{nw})$. Thus we could assign a statistical entropy $4\pi\sqrt{nw}$ to these states.

Given this result it is natural to ask if the same answer can be reproduced by computing the entropy of a black hole carrying the same charge quantum numbers [3 – 6]. The supergravity solution representing this state has vanishing area of the event horizon and hence vanishing entropy. However the curvature and other field strengths grow as we approach the horizon, and hence we expect higher derivative (α') corrections to modify the geometry near the horizon. In contrast the string coupling is small near the horizon and hence we can ignore string loop corretions to leading order. Using the symmetries of the tree level effective action of string theory it was shown, first in [7] for four dimensional black holes and then in [8] for higher dimensional black holes, that for these black holes the α' corrections produce an entropy of the form $a\sqrt{nw}$ where a is a purely numerical constant.[1] This has the same dependence on n and w as the statistical entropy, and is independent of the asymptotic values of various moduli fields as is the case for the statistical entropy. Computation of the numerical coefficient a however requires a detailed knowledge of the higher derivative corrections to the effective action, and was not carried out in these papers.

Recently there has been renewed interest in the problem [11 – 18] due to the observation of Dabholkar that upon inclusion of a special class of higher derivative terms in the effective action of four dimensional heterotic string theory, — obtained by supersymmetrizing the

[1] Some aspects of higher derivative corrections to these solutions have been discussed in [9, 10].

curvature squared term in the action, — the near horizon geometry of the solution becomes $AdS_2 \times S^2$. This gives a finite entropy, and in fact we get exactly the right factor of 4π for the coefficient a appearing in the expression for the black hole entropy. This is in precise agreement with the statistical entropy. The developments which made this computation possible were a series of papers [19 – 28] which supersymmetrized the curvature squared term in the effective action, and used it to compute corrections to the black hole solution/entropy carrying both electric and magnetic charges. The case of elementary string states, which carry only electric charge, then follows from the results of [19 – 28] by setting the magnetic charge to zero. It is however not clear why the other higher derivative corrections which have not been included in the analysis do not affect the result. The general expectation is that there is some kind of non-renormalization theorem that prevents further correction to the entropy formula for the black hole, although no such theorem has been proven to this date.

Given the success of this program for heterotic string compactification to four dimensions, it is natural to ask whether we can extend this analysis to higher dimensions. Here we are at a disadvantage since the construction of a fully supersymmetric version of the action containing curvature squared terms, along the line of [19 – 28], has not been done. Nevertheless one might wonder if it is possible to use the intuition gained from the four dimensional problem to guess the general structure of the horizon of these higher dimensional black holes. This is the problem we undertake in this paper. Our ansatz for the near horizon geometry is $AdS_2 \times S^{D-2}$. This background is characterized by several parameters, — the radii of AdS_2 and S^{D-2}, the constant values of the scalar fields near the horizon, and the flux of the gauge fields through AdS_2. We then demonstrate how, given a classical effective action, we can determine these parameters by solving a set of algebraic equations. We illustrate this procedure by computing all the parameters in a theory where we add to the usual supergravity action the Gauss-Bonnet term [29, 30]. We get a finite area event horizon and finite entropy of the black hole, although the overall numerical factor in the expression for the black hole entropy does not agree with that in the expression for the statistical entropy except in four and five dimensions. Given that we have not even included terms which are related to the Gauss-Bonnet term via supersymmetry transformation, the result is hardly surprising. However the analysis demonstrates that the higher derivative terms do have the capability of modifying the near horizon geometry of these black holes to produce a finite answer for the entropy.

The rest of the paper is organised as follows. We work in units $\hbar = c = 1$ and $\alpha' = 16$. In section 2 we review the arguments of [7, 8, 31] which establish the correct dependence of the entropy of the black hole on the charge quantum numbers n and w characterizing the black hole. In the process of doing so we also restate the arguments of [8] along the lines of [7, 13] that makes explicit the general validity of the argument. In section 3 we propose an ansatz for the modification of the near horizon geometry of the black hole due to α' corrections to the effective action and study its effect on the black hole entropy. In particular we show how our ansatz reduces the problem of finding the near horizon geometry of the black hole solution in the higher derivative theory to solving a set of algebraic equations, and the computation of the entropy associated with the solution to evaluation of an algebraic expression. We also demonstrate this procedure explicitly in the context of an action where we modify the supergravity action by addition of the Gauss-Bonnet term. This gives rise to a regular near horizon solution and finite entropy of the black hole although the numerical coefficient computed from this action does not agree with the one computed from statistical entropy for $D \neq 4, 5$. Finally in section 4 we discuss some aspects of the construction of the full solution that interpolates between the near horizon geometry and the geometry at large radial distance.

2. Supergravity solution for two charge black holes and its near horizon limit

In this section we shall review the black hole solutions of the supergravity equations of motion representing elementary string states in toroidally compactified heterotic string theory, its near horizon limit, and the scaling argument leading to the general form of the entropy associated with these black holes [7, 8].

In order to keep our discussion simple, we shall here consider only a special class of black hole solutions representing a heterotic string wound on a circle. We work with heterotic string theory compactified on $T^n \times S^1$, T^n being an arbitrary n-torus and S^1 being a circle of coordinate radius $\sqrt{\alpha'} = 4$. Let us denote by x^M $(0 \le M \le 9)$ the set of all coordinates, by x^μ $(0 \le \mu \le D-1,\ D = 9-n)$ the coordinates along the non-compact directions, by x^m $(D \le m \le 8)$ the coordinates of T^n and by x^9 the coordinate along S^1. We also denote by $G^{(10)}_{MN}$, $B^{(10)}_{MN}$ and $\Phi^{(10)}$ the ten dimensional string metric, anti-symmetric tensor field and dilaton respectively. For the description of the black hole solution under study we shall only need to consider non-trivial configurations of the fields $G^{(10)}_{\mu\nu}$, $B^{(10)}_{\mu\nu}$, $G^{(10)}_{9\mu}$, $G^{(10)}_{99}$, $B^{(10)}_{9\mu}$ and $\Phi^{(10)}$. We freeze all other field components to trivial background values, and define:[2]

$$\begin{aligned}
\Phi &= \Phi^{(10)} - \frac{1}{2}\ln(G^{(10)}_{99})\,, \qquad S = e^{-\Phi}\,, \qquad T = \sqrt{G^{(10)}_{99}}\,,\\
G_{\mu\nu} &= G^{(10)}_{\mu\nu} - (G^{(10)}_{99})^{-1}\, G^{(10)}_{9\mu}\, G^{(10)}_{9\nu}\,,\\
A^{(1)}_\mu &= \frac{1}{2}(G^{(10)}_{99})^{-1}\, G^{(10)}_{9\mu}\,, \qquad A^{(2)}_\mu = \frac{1}{2}B^{(10)}_{9\mu}\,,\\
B_{\mu\nu} &= B^{(10)}_{\mu\nu} - 2(A^{(1)}_\mu A^{(2)}_\nu - A^{(1)}_\nu A^{(2)}_\mu)\,.
\end{aligned} \tag{2.1}$$

The low energy effective action involving these fields is then given by

$$\begin{aligned}
\mathcal{S} = \frac{1}{32\pi}\int d^Dx\,\sqrt{-\det G}\,S\,\Big[&R_G + S^{-2}\,G^{\mu\nu}\,\partial_\mu S\partial_\nu S - T^{-2}\,G^{\mu\nu}\,\partial_\mu T\partial_\nu T -\\
&-\frac{1}{12}G^{\mu\mu'}G^{\nu\nu'}G^{\rho\rho'}\,H_{\mu\nu\rho}H_{\mu'\nu'\rho'} - T^2\,G^{\mu\nu}\,G^{\mu'\nu'}\,F^{(1)}_{\mu\mu'}F^{(1)}_{\nu\nu'} -\\
&-T^{-2}\,G^{\mu\nu}\,G^{\mu'\nu'}\,F^{(2)}_{\mu\mu'}F^{(2)}_{\nu\nu'}\Big]\,,
\end{aligned} \tag{2.2}$$

[2]Our convention for normalization of the dilaton is the same as that in [7, 8], i.e. e^Φ represents the square of the effective closed string coupling constant.

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where R_G is the scalar curvature computed from the string metric $G_{\mu\nu}$ and

$$\begin{aligned} F^{(a)}_{\mu\nu} &= \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu\,, \qquad a = 1, 2\,, \\ H_{\mu\nu\rho} &= \left[\partial_\mu B_{\nu\rho} + 2\left(A^{(1)}_\mu F^{(2)}_{\nu\rho} - A^{(2)}_\mu F^{(1)}_{\nu\rho}\right)\right] + \text{cyclic permutations of } \mu,\ \nu,\ \rho\,. \end{aligned} \tag{2.3}$$

The overall normalization constant of $1/32\pi$ is completely arbitrary and could be absorbed into a redefinition of S by an multiplicative constant. T has been normalized so that the T-duality transformation takes the form:

$$T \to \frac{1}{T}\,, \qquad F^{(1)}_{\mu\nu} \to F^{(2)}_{\mu\nu}\,, \qquad F^{(2)}_{\mu\nu} \to F^{(1)}_{\mu\nu}\,. \tag{2.4}$$

We shall focus on field configurations for which

$$H_{\mu\nu\rho} = 0\,. \tag{2.5}$$

For studying these configurations we can use the restricted action

$$\begin{aligned} \mathcal{S} = \frac{1}{32\pi}\int d^D x\,\sqrt{-\det G}\,S\Big[&R_G + S^{-2}\,G^{\mu\nu}\,\partial_\mu S\partial_\nu S - T^{-2}\,G^{\mu\nu}\,\partial_\mu T\partial_\nu T \\ &- T^2\,G^{\mu\nu}\,G^{\mu'\nu'}\,F^{(1)}_{\mu\mu'}F^{(1)}_{\nu\nu'} - T^{-2}\,G^{\mu\nu}\,G^{\mu'\nu'}\,F^{(2)}_{\mu\mu'}F^{(2)}_{\nu\nu'}\Big]\,, \end{aligned} \tag{2.6}$$

obtained by setting $H_{\mu\nu\rho} = 0$ in (2.2).

Eq. (2.6) expresses the relevant part of the action in terms of the string metric $G_{\mu\nu}$. The same expression, written in terms of the canonical metric

$$g_{\mu\nu} = S^\gamma\,G_{\mu\nu}\,, \qquad \gamma = \frac{2}{D-2}\,, \tag{2.7}$$

takes the form:

$$\begin{aligned} \mathcal{S} = \frac{1}{32\pi}\int d^D x\,\sqrt{-\det g}\,\Big[&R_g - \frac{1}{(D-2)}\,S^{-2}\,g^{\mu\nu}\,\partial_\mu S\partial_\nu S - T^{-2}\,g^{\mu\nu}\,\partial_\mu T\partial_\nu T - \\ &- S^\gamma T^2\,g^{\mu\nu}\,g^{\mu'\nu'}\,F^{(1)}_{\mu\mu'}F^{(1)}_{\nu\nu'} - S^\gamma T^{-2}\,g^{\mu\nu}\,g^{\mu'\nu'}\,F^{(2)}_{\mu\mu'}F^{(2)}_{\nu\nu'}\Big]\,. \end{aligned} \tag{2.8}$$

With this choice of overall normalization convention the Newton's constant is given by

$$G_N = 2\,. \tag{2.9}$$

Note that this action is invariant under

$$T \to e^\beta T\,, \quad S \to e^\lambda S, \quad A^{(1)}_\mu \to e^{-\beta-\gamma\lambda/2}A^{(1)}_\mu, \quad A^{(2)}_\mu \to e^{\beta-\gamma\lambda/2}A^{(2)}_\mu\,, \quad g_{\mu\nu} \to g_{\mu\nu}\,, \tag{2.10}$$

for arbitrary real numbers β and λ. As we shall argue later, the transformation generated by β is an exact symmetry of the *tree level* string effective action.

We now consider an heterotic string wound w times along the circle S^1 labelled by x^9 and carrying n units of momentum along the same circle. These solutions may be obtained by beginning with the D-dimensional Schwarzschild solution and then by applying a solution generating transformation [32, 33]. The solutions relevant for us are given by [8]

$$
\begin{aligned}
ds^2_{\text{string}} &\equiv G_{\mu\nu} dx^\mu dx^\nu = -(F(\rho))^{-1} \rho^{2\alpha} dt^2 + d\vec{x}^2 \,, \\
\rho^2 &\equiv \vec{x}^2 \,, \qquad F(\rho) \equiv (\rho^\alpha + 2W)(\rho^\alpha + 2N) \,, \qquad \alpha \equiv D - 3 \,, \\
S &= (F(\rho))^{1/2} \, \rho^{-\alpha} \,, \\
T &= \sqrt{\frac{(\rho^\alpha + 2N)}{(\rho^\alpha + 2W)}} \,, \\
A_t^{(1)} &= -\frac{N}{(\rho^\alpha + 2N)} + \text{constant} \,, \\
A_t^{(2)} &= -\frac{W}{(\rho^\alpha + 2W)} + \text{constant} \,,
\end{aligned} \tag{2.11}
$$

where N and W are two arbitrary parameters labelling the solution. The canonical metric associated with this solution is given by

$$
\begin{aligned}
ds^2_c &\equiv g_{\mu\nu} dx^\mu dx^\nu = S^\gamma ds^2_{\text{string}} \\
&= -(F(\rho))^{\frac{\gamma}{2}-1} \rho^{\alpha(2-\gamma)} dt^2 + (F(\rho))^{\frac{\gamma}{2}} \, \rho^{-\gamma\alpha} d\vec{x}^2 \,.
\end{aligned} \tag{2.12}
$$

The solution (2.11) has the property that asymptotically the fields S and T approach 1. We shall be interested in a more general class of solutions for which asymptotically

$$
\begin{aligned}
&g_{\mu\nu} \to \eta_{\mu\nu} \\
&S \to g^{-2} \,, \qquad T \to \frac{R}{4} \,.
\end{aligned} \tag{2.13}
$$

Here g is the value of the D dimensional string coupling and R denotes the asymptotic radius of S^1 measured in the string metric. Such a solution can be obtained from (2.11) by applying the transformation (2.10) with

$$
e^\lambda = g^{-2} \,, \qquad e^\beta = \frac{R}{4} \,. \tag{2.14}
$$

This gives the new solution

$$
\begin{aligned}
ds^2_c &= -(F(\rho))^{\frac{\gamma}{2}-1} \rho^{\alpha(2-\gamma)} \, dt^2 + (F(\rho))^{\frac{\gamma}{2}} \, \rho^{-\gamma\alpha} d\vec{x}^2 \,, \\
\rho^2 &\equiv \vec{x}^2 \,, \qquad F(\rho) \equiv (\rho^\alpha + 2W)(\rho^\alpha + 2N) \,, \qquad \alpha \equiv D - 3 \,, \\
S &= g^{-2} \, (F(\rho))^{1/2} \, \rho^{-\alpha} \,, \\
T &= \frac{R}{4} \sqrt{\frac{(\rho^\alpha + 2N)}{(\rho^\alpha + 2W)}} \,, \\
A_t^{(1)} &= -\frac{4}{R} g^\gamma \, \frac{N}{(\rho^\alpha + 2N)} + \text{constant} \,, \\
A_t^{(2)} &= -\frac{R}{4} g^\gamma \, \frac{W}{(\rho^\alpha + 2W)} + \text{constant} \,.
\end{aligned} \tag{2.15}
$$

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The string metric and the gauge field strengths associated with this solution is given by

$$\begin{aligned} ds^2_{\rm string} &= S^{-\gamma} ds_c^2 = -g^{2\gamma}\,(F(\rho))^{-1} \rho^{2\alpha} dt^2 + g^{2\gamma}\, d\vec{x}^2\,, \\ F^{(1)}_{\rho t} &= \frac{4N}{R}\, g^\gamma\, \frac{\alpha\rho^{\alpha-1}}{(\rho^\alpha + 2N)^2}\,, \\ F^{(2)}_{\rho t} &= \frac{RW}{4}\, g^\gamma\, \frac{\alpha\rho^{\alpha-1}}{(\rho^\alpha + 2W)^2}\,. \end{aligned} \tag{2.16}$$

From the definitions given in (2.1) it follows that the gauge fields $A^{(1)}_\mu$ and $A^{(2)}_\mu$ have as their sources respectively the momentum and winding charge along S^1. Since the asymptotic field configurations $F^{(1)}_{\mu\nu}$ and $F^{(2)}_{\mu\nu}$ are proportional to N and W respectively, N must be proportional to the momentum quantum number n and W must be proportional to the winding number w. We can find the constants of proportionality by comparing the mass of the black hole with that of the elementary string state carrying the same charges. For this we note that the black hole mass M can be read out by using the following asymptotic form of the tt component of the canonical metric:

$$g_{tt} \simeq -1 + \frac{16\pi G_N}{(D-2)\Omega_{D-2}} \frac{M}{\rho^\alpha}\,, \tag{2.17}$$

where

$$\Omega_{D-2} \equiv \frac{2\,\pi^{(D-1)/2}}{\Gamma\left(\frac{D-1}{2}\right)} \tag{2.18}$$

is the volume of the unit $(D-2)$ sphere and G_N is the Newton's constant. Using (2.9) we see that the mass associated with the solution (2.15) is given by:

$$M = \frac{(D-2)(2-\gamma)\,\Omega_{D-2}(N+W)}{32\pi} = \frac{(D-3)\,\Omega_{D-2}(N+W)}{16\pi}\,. \tag{2.19}$$

On the other hand, the mass of the BPS state of an elementary string carrying n units of momentum and w units of winding charge along S^1, measured in the canonical metric, is given by

$$M = g^\gamma \left(\frac{n}{R} + \frac{w\,R}{16}\right). \tag{2.20}$$

Since N and W are expected to be proportional to n and w respectively, comparison of (2.19) and (2.20) yields

$$N = \frac{16\pi}{(D-3)\,\Omega_{D-2}}\, g^\gamma\, \frac{n}{R}\,, \qquad W = \frac{\pi}{(D-3)\,\Omega_{D-2}}\, g^\gamma\, w\, R\,. \tag{2.21}$$

The (singular) horizon of the solution (2.15) is located at $\rho = 0$. It is easy to see that the area of the horizon, measured in the canonical metric $g_{\mu\nu} = S^\gamma\, G_{\mu\nu}$, goes to zero as $\rho \to 0$. Thus the black hole entropy also vanishes in this approximation.

The question that we would like to address is: do higher derivative (α') corrections or string loop corrections modify this result? The effect of string loop corrections is easy to estimate. From (2.15) we see that the square of the effective string coupling, — $S^{-1} = g^2\rho^\alpha/\sqrt{(\rho^\alpha + 2N)(\rho^\alpha + 2W)}$, — remains small everywhere for small g and large-N and

W, i.e. for large-N and w. Thus in the leading approximation we can ignore string loop corrections. In order to analyze the effect of the α' corrections on the near horizon geometry, we focus on the region:

$$\rho^\alpha \ll N, W\,. \tag{2.22}$$

In this region the solution (2.15), (2.16) takes the form:

$$\begin{aligned}
ds^2_{\text{string}} &= -\frac{(D-3)^2\,\Omega^2_{D-2}\,\rho^{2\alpha}}{64\pi^2 nw}\,dt^2 + g^{2\gamma}\,d\vec{x}^2\,,\\
S &= 8\pi g^{-2+\gamma}\,\frac{1}{(D-3)\Omega_{D-2}}\,\rho^{-\alpha}\,\sqrt{nw}\,,\\
T &= \sqrt{\frac{n}{w}}\,,\\
F^{(1)}_{\rho t} &= \frac{(D-3)\Omega_{D-2}}{16\pi n}\,\alpha\,\rho^{\alpha-1}\,,\\
F^{(2)}_{\rho t} &= \frac{(D-3)\Omega_{D-2}}{16\pi w}\,\alpha\,\rho^{\alpha-1}\,.
\end{aligned} \tag{2.23}$$

We now introduce rescaled coordinates:

$$\vec{y} = g^\gamma\,\vec{x}\,,\qquad r = \sqrt{\vec{y}^2} = g^\gamma\,\rho\,,\qquad \tau = g^{-\alpha\gamma}\,\frac{(D-3)\Omega_{D-2}}{4\pi}\,\frac{t}{\sqrt{nw}}\,. \tag{2.24}$$

In this coordinate system (2.23) takes the form:

$$\begin{aligned}
ds^2_{\text{string}} &= -\frac{r^{2\alpha}}{4}\,d\tau^2 + d\vec{y}^2\,,\qquad r^2 \equiv \vec{y}^2\,,\\
S &= \frac{8\pi}{(D-3)\Omega_{D-2}}\,\frac{\sqrt{nw}}{r^\alpha}\,,\\
T &= \sqrt{\frac{n}{w}}\,,\\
F^{(1)}_{r\tau} &= \frac{1}{4}\,\alpha\,r^{\alpha-1}\,\sqrt{\frac{w}{n}}\,,\\
F^{(2)}_{r\tau} &= \frac{1}{4}\,\alpha\,r^{\alpha-1}\,\sqrt{\frac{n}{w}}\,,
\end{aligned} \tag{2.25}$$

where we have used the fact that $\alpha\gamma - 2 + \gamma = 0$ for γ and α given in (2.7) and (2.11) respectively. Notice that in this new coordinate system the solution near the horizon is determined completely by the charge quantum numbers n and w and is independent of the asymptotic values of the moduli g and R. This is an example of the attractor mechanism for supersymmetric black holes [34–36], generalized to higher dimensions.

We now note that the tree level low energy effective action involving charge neutral fields is invariant under a rescaling of the form:

$$G^{(10)}_{99} \to e^{2\beta} G^{(10)}_{99}\,,\qquad G^{(10)}_{9\mu} \to e^{\beta} G^{(10)}_{9\mu}\,,\qquad B^{(10)}_{9\mu} \to e^{\beta} B^{(10)}_{9\mu}\,,\qquad \Phi^{(10)} \to \Phi^{(10)} + \beta\,. \tag{2.26}$$

Physically this corresponds to a rescaling of the compactification radius by e^β (together with a shift in the dilaton to compensate for the effect of the change of the volume of the compact manifold). Clearly the full string theory is sensitive to the radius of compactification and is not invariant under this transformation. However the tree level effective action involving charge neutral fields, which are involved in the construction of the black hole solution, is not sensitive to the compactification radius, and the action as well as all the quantities (e.g. the black hole entropy) computed from the effective action will be unchanged under this rescaling. In terms of the four dimensional fields defined in (2.1) this amounts to:

$$T \to e^\beta T\,, \qquad A^{(1)}_\mu \to e^{-\beta} A^{(1)}_\mu\,, \qquad A^{(2)}_\mu \to e^{\beta} A^{(2)}_\mu\,, \qquad \Phi \to \Phi\,, \qquad G_{\mu\nu} \to G_{\mu\nu}\,. \tag{2.27}$$

Next we recall that the complete tree level effective action of the heterotic string theory in the subsector under study has the form:

$$\mathcal{S} = \int d^D x \sqrt{-\det G}\, S\, \mathcal{L}\Big(G_{\mu\nu}, T, A^{(1)}_\mu, A^{(2)}_\mu, \frac{\partial_\mu S}{S}, \cdots\Big)\,, \tag{2.28}$$

where $\cdots$ stand for derivatives of various fields which have appeared explicitly in the argument of $\mathcal{L}$. Under multiplication of S by a constant, the action gets multiplied by the same constant. This shows that given any solution of the full equations of motion derived from the action (2.28), we can get another solution by the transformation:

$$S \to K\, S \tag{2.29}$$

for an arbitrary constant K, leaving the string metric and other fields unchanged. Since this transformation multiplies the action by a constant factor K, it also multiplies the entropy associated with the black hole by the same factor.

Choosing $e^\beta = \sqrt{w/n}$ in (2.27) and $K = 1/\sqrt{nw}$ in (2.29) we can map the solution (2.25) to:[3]

$$\begin{aligned}
\hat{ds}^2_{\text{string}} &= -\frac{r^{2\alpha}}{4}\, d\tau^2 + d\vec{y}^2\,, \qquad r^2 = \vec{y}^2\,,\\
\hat{S} &= \frac{8\pi}{(D-3)\Omega_{D-2}}\,\frac{1}{r^\alpha}\,,\\
\hat{T} &= 1\,,\\
\hat{F}^{(1)}_{r\tau} &= \frac{1}{4}\,\alpha\, r^{\alpha-1}\,,\\
\hat{F}^{(2)}_{r\tau} &= \frac{1}{4}\,\alpha\, r^{\alpha-1}\,.
\end{aligned} \tag{2.30}$$

[3] We would like to emphasize that the hatted solution is related to the original solution (2.23) by transformations which are exact symmetries of the equations of motion of tree level string theory, but are not exact symmetries of the full string theory.

We now note that

- The solution has no dependence on any parameter and is completely universal.
- (2.30) is an exact solution of the classical low energy supergravity equations of motion. This follows from the fact that (2.15), (2.16) is a solution of these equations for all n and w, and (2.30) is obtained from this solution by taking the limit $n, w \to \infty$ and carrying out operations which are exact symmetries of the classical low energy supergravity equations of motion.
- For $r \gg 1$ the higher derivative corrections to the solution (2.30) are small. This can be seen by introducing a new coordinate η via the relation $\tau = 2\eta/r^\alpha$, and writing the solution as

$$\begin{aligned}
\hat{ds}^2_{\text{string}} &= -d\eta^2 + d\vec{y}^2 + 2\,\alpha\,\frac{\eta}{r}\,d\eta dr - \alpha^2\,\frac{\eta^2}{r^2}dr^2\,, \qquad r^2 \equiv \vec{y}^2\,, \qquad \alpha \equiv D-3\,,\\
\frac{\partial_r \hat{S}}{\hat{S}} &= -\frac{\alpha}{r}\,,\\
\hat{T} &= 1\,,\\
\hat{F}^{(1)}_{r\eta} &= \frac{\alpha}{2\,r}\,,\\
\hat{F}^{(2)}_{r\eta} &= \frac{\alpha}{2\,r}\,.
\end{aligned} \tag{2.31}$$

  From this we see that for fixed η, the metric approaches flat metric and all other fields become trivial for large r. Hence the corrections due to higher derivative terms are small in this region and we expect the solution of the complete classical equations of motion of string theory to be approximated by (2.30) in this limit.

Since (2.30) has a completely universal form without any parameter. and since furthermore the action (2.28) is also completely universal, it is clear that the higher derivative terms in (2.28) will change (2.30) to a universal form:

$$\begin{aligned}
\hat{ds}^2_{\text{string}} &= -e^{2G(\xi)}\,d\tau^2 + d\xi^2 + e^{2F(\xi)}\,d\Omega^2_{D-2}\,,\\
\hat{S} &= e^{\sigma(\xi)-G(\xi)-(D-2)F(\xi)}\,,\\
\hat{T} &= e^{\chi(\xi)}\,,\\
\hat{A}^{(1)}_\tau &= \psi_1(\xi)\,,\\
\hat{A}^{(2)}_\tau &= \psi_2(\xi)\,,
\end{aligned} \tag{2.32}$$

where $d\Omega^2_{D-2}$ denotes the metric on the unit $(D-2)$-sphere and G, F, σ, χ, ψ_1 and ψ_2 are a set of universal functions of the radial coordinate ξ. (2.32) represents a particular parametrization of the most general spherically symmetric configuration; this parametrization has been chosen for later convenience. For large ξ (2.32) must agree with the solution (2.30) for suitable choice of relation between r and ξ. Using the inverse of the transformation (2.29) and (2.27) we can now generate the modified version of the solution (2.25):

$$ds^2_{\text{string}} = -e^{2G(\xi)}\,d\tau^2 + d\xi^2 + e^{2F(\xi)}\,d\Omega^2_{D-2}\,,$$

$$
\begin{aligned}
S &= \sqrt{nw}\, e^{\sigma(\xi)-G(\xi)-(D-2)F(\xi)} \\
T &= \sqrt{\frac{n}{w}}\, e^{\chi(\xi)} \\
A_\tau^{(1)} &= \sqrt{\frac{w}{n}}\, \psi_1(\xi)\,, \\
A_\tau^{(2)} &= \sqrt{\frac{n}{w}}\, \psi_2(\xi)\,.
\end{aligned}
\tag{2.33}
$$

We now turn to the computation of entropy associated with this solution. In the presence of higher derivative corrections the entropy is no longer proportional to the area of the event horizon; there are additional corrections [37–40]. These corrections all have the property that if the action is multiplied by a constant then the entropy associated with a given solution also gets multiplied by the same constant. Now suppose a denotes the entropy associated with the solution (2.32). Then since the solution (2.32) and the action (2.28) are both universal, a must be a purely numerical coefficient. Since the transformation (2.29) with $K = 1/\sqrt{nw}$ multiplies the action by a factor of $1/\sqrt{nw}$, whereas the transformation (2.27) leaves the action invariant, and since upon multiplying the action by a constant factor the entropy associated with the solutions gets multiplied by a constant factor, we see that the entropy associated with the solution (2.33) must be given by:

$$
S_{BH} = a\sqrt{nw}\,. \tag{2.34}
$$

On the other hand counting of states of fundamental heterotic string carrying w units of winding and n units of momentum along S^1 shows that for large-N and w the degeneracy of states grows as $e^{4\pi\sqrt{nw}}$ [1]. Thus the statistical entropy, defined as the logarithm of the degeneracy of states, is given by:

$$
S_{\rm stat} \simeq 4\pi\sqrt{nw}\,, \tag{2.35}
$$

for large-N and w. Thus we see that up to an overall multiplicative constant the statistical entropy agrees with the Bekenstein-Hawking entropy of the black hole.

3. Modification of the solution by higher derivative terms and its near horizon limit

In this section we shall propose an ansatz for how the solution (2.15) is modified by the higher derivative corrections to the effective action close to the horizon of the black hole. In order to simplify the formulæ we shall focus on the modification of the hatted solution (2.30), since once we find the modification of this solution, we can determine the modification of the original solution as in (2.33).

Since the solution in the supergravity limit is invariant under the $(D-1)$-dimensional rotation group $O(D-1)$, we can restrict our search for the modified solution within the class of rotationally invariant configurations. If we now assume that there is some choice of field variables (which need not be the variables which appear directly in the σ-model describing string propagation in this background) in which the horizon acquires a finite size, then it must have the shape of S^{D-2} and we expect the near horizon geometry to be a product of S^{D-2} and a two dimensional space of lorentzian signature. Symmetry considerations do not tell us what this two dimensional space might be; however for $D = 4$ the analysis of [11 − 18] taking into account a special class of higher derivative terms shows that this two dimensional space is AdS_2. We shall assume that even for $D > 4$ the two dimensional space is AdS_2. Since AdS_2 has symmetry group SO(2,1), it is natural to postulate that not only the metric but also the other near horizon field configurations are invariant under this SO(2,1) symmetry. This fixes the form of all the near horizon field configurations up to a few constants:[4]

$$\begin{aligned} \hat{ds}^2_{\rm string} &= v_1 \left(-r^2\, d\tau^2 + \frac{1}{r^2} dr^2\right) + v_2 d\Omega_{D-2}^2\,, \\ \hat{S} &= v_3, \\ \hat{T} &= v_4\,, \\ \hat{F}^{(1)}_{r\tau} &= v_5\,, \\ \hat{F}^{(2)}_{r\tau} &= v_6\,. \end{aligned} \tag{3.1}$$

Here $v_1, \ldots v_6$ are constants to be determined by solving the equations of motion. For consistency we must check that there is a solution that interpolates between (3.1) for small r and (2.30) for large r, but we shall not attempt to analyze this question here. Instead we shall focus on the analysis of the equations of motion near the horizon. Some aspects of the interpolating solution will be discussed in section 4.

First we analyze the equations of motion of the gauge fields. From eq. (2.28) and spherical symmetry of the configuration it follows that these equations take the form:[5]

$$\partial_r \left[\sqrt{-\det G}\, S\, \frac{\partial \mathcal{L}}{\partial F^{(i)}_{r\tau}}\right] = 0\,, \qquad i = 1, 2\,. \tag{3.2}$$

Thus the combination $\left[\sqrt{-\det G}\, S\, \frac{\partial \mathcal{L}}{\partial F^{(i)}_{r\tau}}\right]$ is independent of r. Since for large r the contribution from higher derivative corrections to the action is small, we can evaluate this expression at large r using the supergravity action (2.6) and the form (2.30) for the hatted solution, and then by (3.2) the value of this expression near the horizon must be the same. This gives us two equations:

$$\sqrt{-\det \hat{G}}\, \hat{S}\, \frac{\partial \hat{\mathcal{L}}}{\partial \hat{F}^{(i)}_{r\tau}} = \frac{1}{2\Omega_{D-2}} \sqrt{\det h^{(D-2)}}, \qquad \text{for } i = 1, 2\,, \tag{3.3}$$

where $h^{(D-2)}_{ij}$ denotes the metric on a unit $(D-2)$-sphere, and $\hat{\mathcal{L}}$ denotes $\mathcal{L}$ evaluated for the hatted solution.

[4]Note that the 'string metric' does not necessarily refer to the metric that appears in the σ-model describing string propagation in this background. Rather it simply implies that the metric appearing in (3.1) has the same scaling properties as the string metric under the transformations (2.27), (2.29).

[5]This form of the equations of motion requires that the lagrangian density depends only on the gauge field strengths and not for example on the gauge fields themselves. Thus inclusion of Chern-Simons forms in the action would modify this structure. Since for the solution under study the anti-symmetric three form field strength $H_{\mu\nu\rho}$ vanishes, we can use the form of the equation given in (3.2).

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Let us define the function $f(v_1, \ldots, v_6)$ through the relation:

$$\sqrt{-\det \hat{G}}\, \hat{S}\, \mathcal{L}\Big(\hat{G}_{\mu\nu}, \hat{T}, \hat{A}_\mu^{1)}, \hat{A}_\mu^{(2)}, \frac{\partial_\mu \hat{S}}{\hat{S}=0}\Big)\Big|_{r\simeq 0} = \frac{1}{\Omega_{D-2}}\sqrt{\det h^{(D-2)}}\, f(v_1, \ldots, v_6)\,, \quad (3.4)$$

where the hatted configuration is as given in (3.1). Then eqs. (3.3) may be written as:

$$\frac{\partial f}{\partial v_5} = \frac{1}{2}\,, \qquad \frac{\partial f}{\partial v_6} = \frac{1}{2}\,. \quad (3.5)$$

We now turn to the equations of motion for the metric and the scalar fields S and T. Due to the $\mathrm{SO}(2,1)\times\mathrm{SO}(D-1)$ symmetry of the configuration, the metric equation reduces to two independent scalar equations; obtained by extremizing the action with respect to any one of the components of the metric along AdS_2, and any one of the components of the metric along S^{D-2}. The scalar field equations are obtained by extremizing the action with respect to these fields. The resulting set of equations may be written as[6]

$$\frac{\partial f}{\partial v_i} = 0\,, \qquad 1 \le i \le 4\,. \quad (3.6)$$

Eqs. (3.5) and (3.6) give complete set of equations which need to be solved in order to compute the parameters v_i.

Given the solution, we can compute the entropy associated with the solution by the formula given in [37−40]. In the present context this formula gives:

$$\hat{S}_{BH} = 8\pi\, \hat{S}\, \frac{\partial \hat{\mathcal{L}}}{\partial \hat{R}_{Grtrt}} \hat{G}_{rr}\hat{G}_{tt}\hat{A}_{D-2}\Big|_{r=0}\,, \quad (3.7)$$

where $\hat{R}_{G\mu\nu\rho\sigma}$ denotes the Riemann tensor computed using the string metric $\hat{G}_{\mu\nu}$,[7] $\hat{A}_{D-2}$ is the area of the event horizon measured in the string metric $\hat{G}_{\mu\nu}$, and the hat on top of S_{BH} denotes that we are computing the entropy associated with the hatted solution. Thus $\hat{S}_{BH}$ can be identified with the constant a appearing in (2.34). For the background (3.1)

$$\hat{A}_{D-2} = (v_2)^{(D-2)/2}\Omega_{D-2}\,, \qquad \hat{G}_{rr} = \frac{v_1}{r^2}\,, \qquad \hat{G}_{tt} = -v_1 r^2\,, \qquad \hat{S} = v_3\,, \quad (3.8)$$

and we have

$$a = \hat{S}_{BH} = -8\pi\,(v_1)^2\,(v_2)^{(D-2)/2}\, v_3\, \Omega_{D-2}\, \frac{\partial \hat{\mathcal{L}}}{\partial \hat{R}_{Grtrt}}\Big|_{r=0}\,. \quad (3.9)$$

This expression can be simplified using eq. (3.4). If we define $f(\lambda; v_1, \cdots v_6)$ through a similar equation by multiplying in the expression for $\mathcal{L}$ every factor of $\hat{R}_{G\alpha\beta\gamma\delta}$ for $\alpha,\beta,\gamma,\delta = r,t$ by a factor of λ, then we have the relation:

$$\sqrt{-\det \hat{G}}\, \hat{S}\, \hat{R}_{G\alpha\beta\gamma\delta}\, \frac{\partial \hat{\mathcal{L}}}{\partial \hat{R}_{G\alpha\beta\gamma\delta}}\Big|_{r\simeq 0} = \frac{1}{\Omega_{D-2}}\sqrt{\det h^{(D-2)}}\, \frac{\partial f(\lambda; v_1, \ldots v_6)}{\partial \lambda}\Big|_{\lambda=1}\,. \quad (3.10)$$

[6] This is possible because we have chosen the most general configuration consistent with the $\mathrm{SO}(2,1)\times\mathrm{SO}(D-1)$ symmetry. Furthermore, in the coordinate system that we have chosen, the symmetry transformation laws are independent of the parameters $\{v_i\}$.

[7] We could use any other metric, e.g. the canonical metric for this computation as long as we use the same metric everywhere in eq. (3.7). In computing the derivative in (3.7) we regard different components of the Riemann tensor as independent variables.

Using the relation:

$$R_{G\alpha\beta\gamma\delta} = -v_1^{-1}\left(G_{\alpha\gamma}G_{\beta\delta} - G_{\alpha\delta}G_{\beta\gamma}\right), \qquad \alpha,\beta,\gamma,\delta = r,t\,, \tag{3.11}$$

and the expression for $G_{\mu\nu}$ given in eqs. (3.1), we can rewrite (3.10) as

$$\left.\frac{\partial\hat{\mathcal{L}}}{\hat{R}_{Grtrt}}\right|_{r\simeq 0} = \frac{1}{4}\,v_1^{-2}\,v_2^{-(D-2)/2}\,v_3^{-1}\,\frac{1}{\Omega_{D-2}}\,\left.\frac{\partial f(\lambda;v_1,\dots v_6)}{\partial\lambda}\right|_{\lambda=1}. \tag{3.12}$$

Eq. (3.9) now gives

$$a = -2\pi\,\left.\frac{\partial f(\lambda;v_1,\dots v_6)}{\partial\lambda}\right|_{\lambda=1}. \tag{3.13}$$

In order to proceed further we need to know the explicit form of $\mathcal{L}$. This however is not known. What we plan to do next is to consider a special higher derivative correction to the action which is known to exist in the tree level heterotic string theory and study its effect on the solution under consideration. This is the Gauss-Bonnet term [29, 30]:[8]

$$\Delta S = \frac{C}{16\pi}\int d^Dx\,\sqrt{-\det G}\,S\,\left[R_{G\mu\nu\rho\sigma}R_G^{\mu\nu\rho\sigma} - 4R_{G\mu\nu}R_G^{\mu\nu} + R_G^2\right] \tag{3.14}$$

where C is a constant. For heterotic string theory

$$C = 1 \tag{3.15}$$

but we shall work with arbitrary C so that we can identify the effect of the higher derivative term by examining the C dependence of the final formulæ. Combining (2.6), (3.14) and comparing this with (2.28), we get

$$\begin{aligned}\mathcal{L} = \frac{1}{32\pi}\Big[& R_G + S^{-2}\,G^{\mu\nu}\,\partial_\mu S\partial_\nu S - T^{-2}\,G^{\mu\nu}\,\partial_\mu T\partial_\nu T - \\ & - T^2\,G^{\mu\nu}\,G^{\mu'\nu'}\,F^{(1)}_{\mu\mu'}F^{(1)}_{\nu\nu'} - T^{-2}\,G^{\mu\nu}\,G^{\mu'\nu'}\,F^{(2)}_{\mu\mu'}F^{(2)}_{\nu\nu'} + \\ & + 2\,C\left\{R_{G\mu\nu\rho\sigma}R_G^{\mu\nu\rho\sigma} - 4R_{G\mu\nu}R_G^{\mu\nu} + R_G^2\right\}\Big] + \cdots\,,\end{aligned} \tag{3.16}$$

where ... denotes other terms which we are not including in our analysis. These include terms related to (3.14) by supersymmetry as well as other higher derivative terms.

Substituting (3.1) into (3.16), and using the definition of $f(\vec{v})$ given in eq. (3.4), we get

$$\begin{aligned}f(v_1,\dots v_6) = \frac{\Omega_{D-2}}{32\pi}\,v_1\,v_2^{(D-2)/2}\,v_3\Big[& -\frac{2}{v_1} + \frac{(D-2)(D-3)}{v_2} + \frac{2\,v_4^2\,v_5^2}{v_1^2} + \frac{2\,v_6^2}{v_4^2\,v_1^2} + \\ & + \frac{2\,C}{v_2^2}\,(D-2)(D-3)(D-4)(D-5) - \\ & - \frac{8C}{v_1 v_2}(D-2)(D-3)\Big].\end{aligned} \tag{3.17}$$

[8] Effect of Gauss-Bonnet term on black hole entropy has been studied earlier in different context [41, 42].

The solutions to (3.5), (3.6) are now given by

$$\begin{aligned} v_2 &= 4\,C\,[(D-2)(D-3)-(D-4)(D-5)]\,, \\ v_1 &= \frac{2\,v_2}{(D-2)(D-3)}\,, \\ \widetilde{v}_3 \equiv \frac{\Omega_{D-2}}{32\pi}\, v_1\, v_2^{(D-2)/2}\, v_3 &= \left[\frac{16}{v_1^3 v_2}(v_2 + 4\,C\,(D-2)(D-3))\right]^{-1/2}\,, \\ v_4 &= 1\,, \\ v_5 = v_6 &= \frac{v_1^2}{8\widetilde{v}_3}\,. \end{aligned} \tag{3.18}$$

Finite values of v_1, v_2 and v_3 shows that the inclusion of the higher derivative terms (3.14) into the action does stretch the horizon of the black hole.

Finally we turn to the computation of entropy using (3.13). For the lagrangian density (3.16), $f(\lambda; v_1, \cdots v_6)$ is given by[9]

$$\begin{aligned} f(\lambda; v_1, \ldots, v_6) = \frac{\Omega_{D-2}}{32\pi}\, v_1\, v_2^{(D-2)/2}\, v_3 \Bigg[&-\frac{2\lambda}{v_1} + \frac{(D-2)(D-3)}{v_2} + \frac{2\,v_4^2\,v_5^2}{v_1^2} + \frac{2\,v_6^2}{v_4^2\,v_1^2} + \\ &+ \frac{2\,C}{v_2^2}\,(D-2)(D-3)(D-4)(D-5) - \\ &- \frac{8C\lambda}{v_1 v_2}(D-2)(D-3)\Bigg]\,. \end{aligned} \tag{3.19}$$

Eqs. (3.13) and (3.18) now give

$$\begin{aligned} a &= 4\pi \frac{\widetilde{v}_3}{v_1\, v_2}\,(v_2 + 4\,C\,(D-2)\,(D-3))\,, \qquad (3.20) \\ &= \pi\,\sqrt{C}\,\sqrt{\frac{2}{(D-2)(D-3)}}\,\{8(D-2)(D-3) - 4(D-4)(D-5)\}^{1/2}\,, \quad C = 1\,. \end{aligned}$$

This differs from the expected value 4π calculated using the statistical entropy except for $D = 4$ and $D = 5$.[10] This is not a surprise since unlike in [11 − 18] our analysis does not include the complete set of terms needed to supersymmetrize the action; – the agreement with the correct answer for $D = 4$ and $D = 5$ is most likely an accident. It is however encouraging that the effect of part of the higher derivative terms do modify the geometry so as to yield a finite value of the entropy of these black holes.

Before concluding this section we would like to mention some subtle points in the analysis.

1. First of all note that the gauge field fluxes, encoded in the non-zero constants appearing on the right hand side of eqs. (3.5), are crucial for getting a non-trivial solution of the equations of motion. If the right hand sides of eqs. (3.5) had been zero, we

[9] $f(\lambda, \vec{v})$ can be obtained from $f(\vec{v})$ by the rescaling $v_1 \to \lambda^{-1} v_1$, $v_2 \to v_2$, $\widetilde{v}_3 \to \widetilde{v}_3$, $v_4 \to v_4$, $v_5 \to \lambda^{-1} v_5$ and $v_6 \to \lambda^{-1} v_6$.

[10] The agreement for $D = 4$ is probably related to the result of [41] where a similar agreement was found for large black holes carrying both electric and magnetic charges.

would get $v_5 = v_6 = 0$. Eqs. (3.6) with f given in (3.17) would then imply that v_1 and v_2 must be infinite. This corresponds to the flat space limit.

2. By examining the structure of the solution (3.1) one would be tempted to conclude that string propagation in this background is described by a direct sum of two conformal field theories (CFT); — one associated with the r, τ coordinates labelling AdS_2 and the other associated with the angular coordinates labelling S^{D-2}. The background gauge fields only affect the CFT associated with the r, τ coordinate since only the r, τ component of the gauge fields are non-zero. This would give rise to a coupling between the CFT associated with the r, τ coordinate and the CFT associated with the compact circle S^1. Had this picture been correct, the equation determining the radius of S^{D-2} would be governed by the requirement of conformal invariance of the two dimensional field theory involving the angular coordinates, and hence would not involve the gauge fields. But we have just argued that if the gauge fields vanish then there is no non-trivial solution for v_2, and so the equation determining the radius of S^{D-2} does involve the gauge fields. The resolution of this puzzle lies in the fact that the metric $G_{\mu\nu}$ appearing in the solution (3.1) need not be the same metric that appears in the σ-model describing string propagation in this background; instead two metrics may be related by complicated field redefiniton. This point has already been emphasized earlier in footnote 4.

4. On the interpolating solution

In this section we shall make a few remarks about the construction of the full solution that interpolates between the solution (3.1) near the horizon and (2.30) for large r. The general form of the solution is given in (2.32). The large r solution (2.30) corresponds to the choice

$$r = \xi\,, \quad e^G = \frac{r^\alpha}{2}\,, \quad e^F = r\,, \quad e^\sigma = \frac{4\pi r^{D-2}}{(D-3)\Omega_{D-2}}\,, \quad \chi = 0\,, \quad \psi_1 = \psi_2 = \frac{r^\alpha}{4} + \text{constant}\,. \tag{4.1}$$

in eq. (2.32). On the other hand the small r solution (3.1) corresponds to the choice

$$\begin{aligned} &r = e^{\xi/\sqrt{v_1}}\,, \quad e^G = \sqrt{v_1} r\,, \quad e^F = \sqrt{v_2}\,, \quad e^\sigma = v_1^{1/2} v_2^{(D-2)/2} v_3\, r\,, \\ &\chi = \ln v_4\,, \quad \psi_1 = v_5\, r + \text{constant}\,, \quad \psi_2 = v_6\, r + \text{constant}\,. \end{aligned} \tag{4.2}$$

Our aim is to find a solution that interpolates between (4.1) for large r and (4.2) for small r.[11] For this we can substitute the general form of the solution (2.32) into the action given in (2.28), (3.16). After performing the angular integration and some integration by parts in the ξ variable this gives:[12]

$$\mathcal{S} = \int dt\, d\xi\, \widetilde{\mathcal{L}}\,,$$

[11] For a recent discussion on the subtleties in constructing black hole solutions in the presence of higher derivative terms, see [43].

[12] For studying α' corrections to the D-brane solutions such reduction techniques have been used earlier in [44].

$$
\begin{aligned}
\widetilde{\mathcal{L}} = \frac{\Omega_{D-2}}{32\pi}\, e^{\sigma} \Bigg[& -(G')^2 - (D-2)\,(F')^2 + (D-2)(D-3)e^{-2F} + (\sigma')^2 - \\
& -(\chi')^2 + 2\,e^{2(\chi-G)}\,(\psi_1')^2 + 2\,e^{-2(\chi+G)}\,(\psi_2')^2 + +8C(D-2)(D-3)\times \\
& \times \Big\{ ((F')^2 - e^{-2F})((G')^2 + (D-2)F'G' - \sigma' G') - \\
& \quad - \frac{1}{3}(D-4)\sigma'(F')^3 + (D-4)\sigma' F' e^{-2F} + \\
& \quad + (D-4)(F')^2((F')^2 - 3\,e^{-2F}) + \frac{1}{4}(D-4)(D-5)((F')^2 - e^{-2F})^2 \Big\} \Bigg] .
\end{aligned}
\tag{4.3}
$$

Here $'$ denotes derivative with respect to ξ. Note that the action does not contain any term involving two or more derivatives of the fields. This is a consequence of the fact that we have chosen the curvature squared terms in the Gauss-Bonnet form [29]. The implication of this to the present problem will be discussed shortly.

Besides having to satisfy the equations of motion derived from the action (4.3), the fields F, G, σ, χ, ψ_1 and ψ_2 must also satisfy the constraints associated with the residual gauge symmetry $\xi \to \xi$+constant in the parametrization used in eq. (2.32). This amounts to setting the $\xi\xi$ component of the energy momentum tensor, computed from the action (4.3), to zero. In terms of the lagrangian density $\widetilde{\mathcal{L}}$ defined in (4.3), the constraint equation takes the form:

$$
F'\,\frac{\partial\widetilde{\mathcal{L}}}{\partial F'} + G'\,\frac{\partial\widetilde{\mathcal{L}}}{\partial G'} + \sigma'\,\frac{\partial\widetilde{\mathcal{L}}}{\partial \sigma'} + \chi'\,\frac{\partial\widetilde{\mathcal{L}}}{\partial \chi'} + \sum_{i=1}^{2}\psi_i'\,\frac{\partial\widetilde{\mathcal{L}}}{\partial \psi_i'} - \widetilde{\mathcal{L}} = 0\,. \tag{4.4}
$$

It is easy to check that for the v_i's given in (3.18), (4.2) gives an exact solution to the equations of motion derived from the action (4.3) and the constraint equation (4.4). On the other hand (4.1) is an approximate solution to these equations for large ξ where the effect of the terms proportional to C is negligible. We need to find a solution to the equations of motion for F, G, σ, χ, ψ_1 and ψ_2 derived from this action that interpolates between (4.1) for large ξ and (4.2) for small ξ. First we can try to carry out some consistency checks by constructing the Noether currents associated with various symmetries of the problem and checking if the conservation laws are compatible with our proposal for the near horizon geometry. In particular, since the conservation law implies that the ξ component of the current should be ξ independent, we can compare the ξ components of the conserved currents for the configurations (4.1) and (4.2) to see if they agree. Unfortunately this does not lead to any non-trivial check. For example the conservation laws associated with the shift symmetry of ψ_1 and ψ_2 are satisfied due to (3.5), while the conservation law associated with the ξ translation follows trivially from the fact that the $\xi\xi$ component of the stress tensor vanishes at both ends due to (4.4). Finally the conservation law associated with the $G \to G + \lambda$, $\psi_i \to e^{\lambda}\psi_i$ symmetry can be satisfied by adjusting the constant terms in the expression for the ψ_i's in eqs. (4.1), (4.2).

We now turn to the explicit analysis of the equations of motion and the constraint equations. The analysis may be simplified by noting that the action (4.3) has a symmetry under $\chi \to -\chi$, $\psi_1 \leftrightarrow \psi_2$. Since both the large and small ξ solutions are invariant under

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this transformation, we can restrict ourselves to the symmetric configuration:

$$\chi = 0\,, \qquad \psi_1 = \psi_2\,. \tag{4.5}$$

The χ equation is now trivially satisfied. ψ_1 and ψ_2 equations give:

$$e^{\sigma-2G}\,\psi_1' = e^{\sigma-2G}\,\psi_2' = \text{constant} = \frac{4\pi}{\Omega_{D-2}}\,, \tag{4.6}$$

where the value of the constant has been fixed by studying the solution at large ξ.

The non-trivial equations are those of G, F and σ. Each of them gives a second order differential equation by virtue of the fact that the action contains only terms first order in the derivatives. Thus the number of integration constants is the same as that in the absence of higher derivative corrections, and spurious oscillations seen in the interpolating solution at large radius [13, 14] are absent here. Nevertheless we need six integration constants (of which one can be eliminated due to the constraint (4.4)) and it is not *a priori* clear that for a suitable choice of these integration constants we get a solution that interpolates between the two limiting forms (4.1) and (4.2).

In order to gain some insight into the problem we shall analyze the linearized equations of motion around the background (4.1) for large r since a similar analysis turned out to be instructive in case of four dimensional string theories [13]. For this we introduce fluctuations g, f and $\widetilde{\sigma}$ around this background through the equations:

$$e^G = \frac{r^\alpha}{2}\,e^g\,, \qquad e^F = r\,e^f\,, \qquad e^\sigma = \frac{4\pi r^{D-2}}{(D-3)\Omega_{D-2}}\,e^{\widetilde{\sigma}}\,, \qquad \chi = 0\,, \tag{4.7}$$

with ψ_1' and ψ_2' given by eq. (4.6). The linearized equations of motion for the fluctuations g, f and $\widetilde{\sigma}$, derived from the action (4.3), then take the form:

$$\begin{aligned} r^2 g'' + (\alpha+1) r g' - 2\alpha^2 g + \alpha r \widetilde{\sigma}' + 2\alpha^2\widetilde{\sigma} &= \mathcal{O}(C)\,, \\ r^2 f'' + (\alpha+1) r f' + 2\alpha f + r\widetilde{\sigma}' &= \mathcal{O}(C)\,, \\ \alpha r g' - \alpha^2 g + (\alpha+1) r f' + \alpha(\alpha+1) f + r^2\widetilde{\sigma}'' + (\alpha+1) r\widetilde{\sigma}' + \alpha^2\widetilde{\sigma} &= \mathcal{O}(C)\,, \end{aligned} \tag{4.8}$$

where $\mathcal{O}(C)$ denotes the contribution from the Gauss-Bonnet term in the action. On the other hand the constraint (4.4) takes the form:

$$-\,\alpha\, r\, g' + \alpha^2\, g - (\alpha+1)\, r\, f' + \alpha\,(\alpha+1)\, f + (\alpha+1)\, r\,\widetilde{\sigma}' - \alpha^2\,\widetilde{\sigma} = \mathcal{O}(C)\,. \tag{4.9}$$

We shall first solve these equations ignoring these higher derivative terms and then argue that their effect is small. For $C = 0$ the six linearly independent solutions of the equations of motion (4.8) are found to be:

$$g = 1\,, \qquad f = 0\,, \qquad \widetilde{\sigma} = 1\,, \tag{4.10}$$

$$g = \alpha\, r^{-1}\,, \qquad f = r^{-1}\,, \qquad \widetilde{\sigma} = (\alpha+1)\, r^{-1}\,, \tag{4.11}$$

$$g = r^\alpha\,, \qquad f = 0\,, \qquad \widetilde{\sigma} = 0\,, \tag{4.12}$$

$$g = r^{-\alpha}\,, \qquad f = r^{-\alpha}\,, \qquad \widetilde{\sigma} = 2\, r^{-\alpha}\,, \tag{4.13}$$

$$g = \frac{3\alpha-1}{3\alpha}\, r^{-2\alpha}\,, \qquad f = \frac{1}{(\alpha+1)}\, r^{-2\alpha}\,, \qquad \widetilde{\sigma} = r^{-2\alpha}\,, \tag{4.14}$$

$$g = \frac{\alpha}{2\alpha-1}\, r^{-\alpha+1}\,, \qquad f = \frac{\alpha-1}{\alpha+1}\, r^{-\alpha+1}\,, \qquad \widetilde{\sigma} = r^{-\alpha+1}\,. \tag{4.15}$$

The constraint equation (4.9) eliminates the solution (4.15). Since for each solution the fields g, f and $\widetilde{\sigma}$ have a power law behaviour r^β for some β, the higher derivatives terms involving these fields are suppressed by powers of $1/r$ for large r. This justifies neglecting the correction terms proportional to C in this region.

A generic solution of the equations of motion and the constraint equation near the background (4.1) will be given by an arbitrary linear combination of the five solutions given in (4.10)–(4.14). Some of these fluctuations can be recognized as familiar objects. For example (4.10) corresponds to a scaling of the time coordinate τ and (4.11) corresponds to a shift in the radial variable r. Thus these modes are gauge artifacts. (4.12) represents the result of keeping correction terms of order ρ^α in going from (2.15), (2.16) to (2.23) (with $n = w$, $R = 4$). Since the original solution (2.15), (2.16) preserves half of the space-time supersymmetry, it follows that the deformation (4.12) preserves half of the space-time supersymmetry. The other two modes (4.13), (4.14) are not familiar objects; however note that each of these modes decays for large r. Also we expect that these deformations do not preserve any space-time supersymmetry.

We now propose the following scenario for constructing the interpolating solution. We deform the near horizon geometry (4.2) by appropriate set of perturbations in the fields G, F and σ which are non-singular as $r \to 0$ but grows as r increases. As we evolve the solution towards large r, we expect that the solution will approach (4.1) plus some appropriate deformations described in (4.10)–(4.14). Of these (4.10) and (4.11) may be removed by scaling of τ and shift of r, whereas (4.13) and (4.14) vanish for large r. As a result we only need to worry about the deformation (4.12). As already pointed out, this is a physical deformation of the background and should be present in the full solution.[13] Thus in order that our ansatz for the near horizon geometry is consistent, the non-singular deformations of the background (4.2) should contain a set of parameters such that by adjusting these parameters we can adjust the coefficient of the deformation proportional to (4.12) for large r. In particular for a suitable choice of parameters this coefficient may be made to vanish, and we should recover the configuration (4.1) for large r.

In principle we could analyze the non-singular deformations of the background (4.2) for small r and determine if there are enough parameters which allow us to adjust the coefficient of (4.12). However since the curvature and the other field strengths in (4.2) are of the order of the string scale, this analysis is sensitive to the precise form of the higher derivative terms, and could change when we add to the action other terms needed to satisfy the requirement of space-time supersymmetry. For this reason we shall not carry out the explicit analysis of this problem here. We would like to note however that had we started with the fully supersymmetric action, we could have tried to find the interpolating solution by imposing the half-BPS condition. Typically this would simplify the analysis since the BPS conditions give first order equations for various fields instead of second order equations. The construction of the full solution would involve deforming the near horizon geometry (4.2) by a half-BPS perturbation that is non-singular as $r \to 0$ but

[13] This deformation helps us get out of the $1/r^\alpha$ dependence of the field S as in (2.25) and makes it approach a constant value for large r as in (2.15).

grows at large r, and showing that for an appropriate choice of perturation parameters the geometry approaches that given in (4.1) for large r. In fact in this case the deformations given in (4.13)–(4.14) will be absent from the very beginning as they do not satisfy the requirement of preserving half of the space-time supersymmetry.

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